Optimal Investment Strategy of an Informed Trader in the Presence of a Positive Market Shock

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1 Background

In the context of a financial market, informed traders are those that possess intelligent information that is not generally available to the public. This information can result in a trading strategy that would presumably provide higher returns than that available to the public. Insider trading has been studied before by [Kyl85], [ST08], [JMZ03], [BCW00]. Kyle [Kyl85] considers the case of a single risk-neutral insider that aims to maximize profit in the presence of noise traders. Back et al. [BCW00] study the optimal trading strategy with more than one informed trader and discuss how the correlation between the traders' information can cause imperfect competition among them. Jeng et al. [JMZ03] evaluate the performance of insider trading in order to estimate their returns.

2 Basic Problem

In this project, we study the case of a single-stock, finite-horizon, informed trader who aims to maximize his expected trading profit. The optimal execution strategy is the policy that maximizes the expected profit and specifies the amounts to be traded on each trading period. Furthermore, we assume that the trader has private information about the price of the stock at a specified future date, T.

We assume trading takes place during T discrete time intervals. Denote the number of shares purchased at time t by u_t, for $t = 1, \dots, T$. At each time, $t = 1, \dots, T$, the price of the security is p_t and the transaction cost per unit bought or sold is κ. Thus the amount spent at time t is $u_t p_t + \kappa |u_t|$. The overall cost is equal to

$$\Pi = p^T u + \kappa \|u\|_1$$

The problem data are the the initial price p_0, k, and T, where $p_T = k p_0$. We also assume that the trader is forced to liquidate at time T, *i.e.*, $\mathbf{1}^T u = 0$.

The stock price p_t, $t = 1, \ldots, T-1$ is unknown. In this problem, we assume that the path of price of the stock from time $t = 1, \ldots, T$ takes one of N equiprobable scenarios, all of which have the final price $p_T = kp_0$. Thus, $\mathbf{p}^{(j)} = (p_1^{(j)}, \ldots, p_T^{(j)})$ is the jth possible scenario. We also assume that the trading volume u is constrained by a convex set $\mathcal{U}$. More formally, the optimization problem is:

$$\begin{array}{ll} \text{minimize} & \frac{1}{k}\sum_{j=1}^{N} \mathbf{p}^{(j)T} u + \kappa \|u\|_1 \\ \text{subject to} & \mathbf{1}^T u = 0 \\ & u \in \mathcal{U}. \end{array} \qquad (1)$$

3 The Feasibility Set, $\mathcal{U}$

Several factors limit the trader's actions. These are represented by the convex set $\mathcal{U}$. Existing literature considers the impact of trading volume on market price [BL98], [Kyl85]. This property can be incorporated into $\mathcal{U}$ by limiting the maximum trading volume in each given day. The trader may also be constrained by budget; for example, the maximum total long position must be less than $\overline{B}$, and the maximum total short position must be less than $\underline{B}$.

The following example is a convex description for $\mathcal{U}$ that encapsulates each of these examples.

$$\mathcal{U} = \{\mathbf{u} : |u_t| \le U^{max}, t = 1, \ldots, T-1, \mathbf{1}^T u_+ \le \overline{B}, \mathbf{1}^T u_- \le \underline{B}\}$$

where $\underline{B}$, $\overline{B}$ are constants, and $u_+ = \max(u, 0)$, and $u_- = \min(u, 0)$.

4 Numerical Results

We considered a problem instance with $T = 60$, $k = 3$, $p_0 = 1$ as problem data, and used $N = 10000$, $\mu = 0$, $\sigma = 0.1$ to generate N price path scenarios using geometric brownian motion. The initial price p_0 and the final price $p_T = kp_0$ were set deterministically for each of the N possible price paths. We used the constraints specified by $\mathcal{U}$ in §3, with the parameter values $\kappa = 0.01$, $U^{max} = 100$, $U^{min} = -100$, $\overline{B} = 1000$, $\underline{B} = -1000$.

We calculated the optimal trading strategy u^* for this problem instance. It is shown in figure 1.

A histogram of the profits for all N price trajectories is depicted in figure 2. We also tested the performance of the strategy by generating a new set of N price trajectories using the same distribution as the initial price set. The histogram of the profit resulting from adopting u^* across the second set of price trajectories is also provided in the bottom plot in Figure 2.

We observe that although negative profit is possible, in most of the possible price trajectories the expected profit is \$199 and its standard deviation is \$85.6. Moreover, by adopting u^* on the validation price trajectory set, similar profits are attained. The expected profit is \$199 and its standard deviation is \$90.

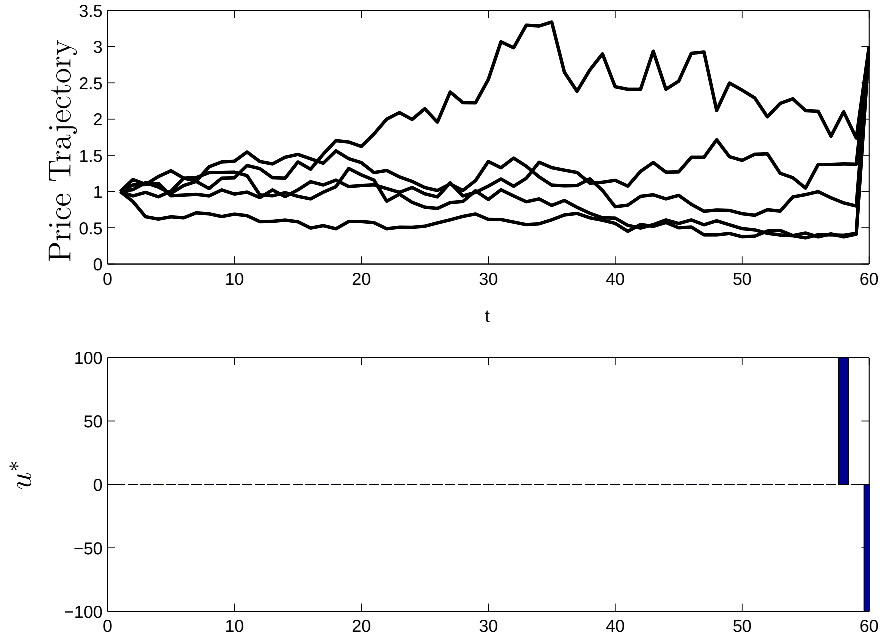

Figure 1: Some example price trajectories (top), and trader's position resulting from adopting the optimal trading strategy, u^*(bottom). Since the price from time $t = 1, \ldots, T-1$ is relatively constant, the trader delays purchasing shares until the day before the shock.

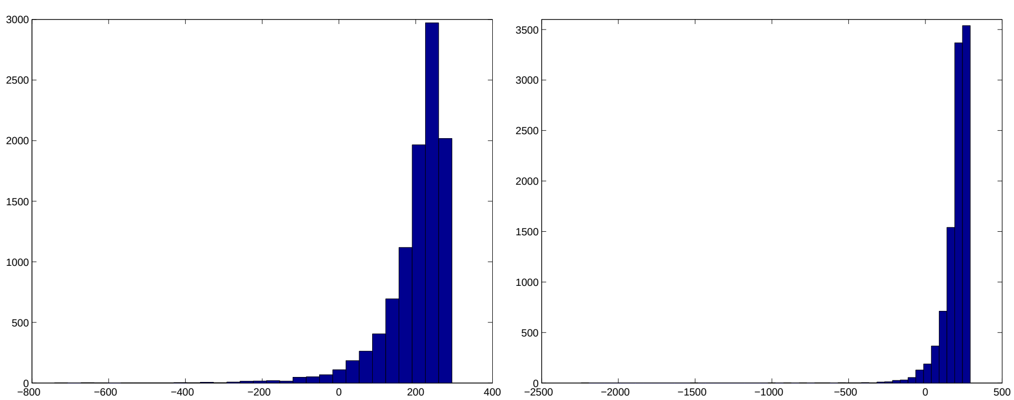

Figure 2: Profit histogram for adopting the designed strategy u^* on the original N price path scenarios (left), and validating on a a different set of N price path scenarios generated using the same distribution (right).

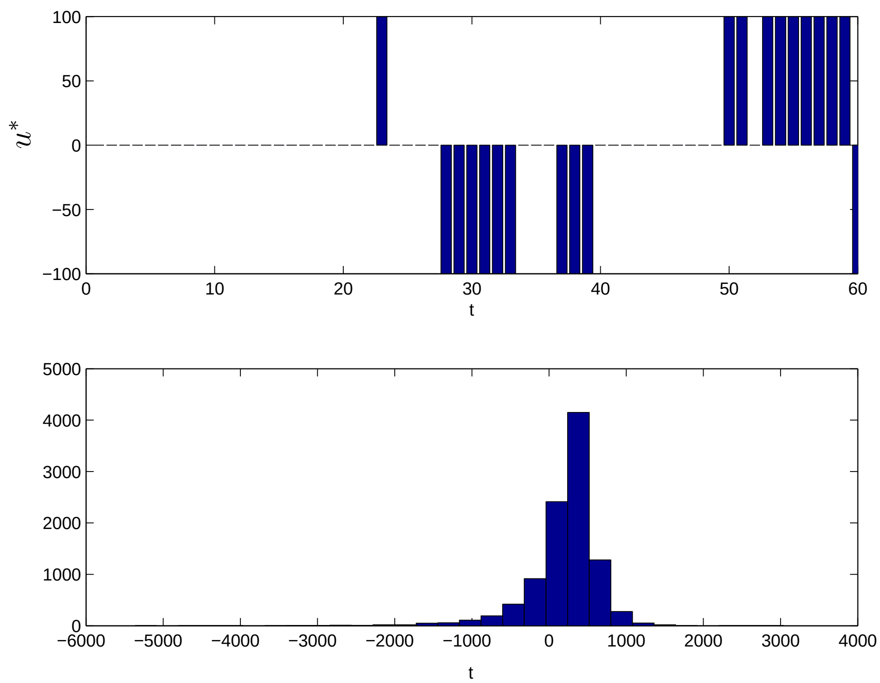

Figure 3: Optimal trading strategy for the case of zero trading costs, *i.e.*, $\kappa = 0$ (top) and the corresponding profit histogram(bottom). Since the trading cost is zero, the trader trades more often to take advantage of the lower prices.

4.1 Zero Trading Cost

Figure 3 illustrates the optimal trading strategy for the case of zero trading cost, *i.e.*, $\kappa = 0$ and its corresponding profit histogram. As a result, the trader trades more frequently.

4.2 Comparison with Alternative Strategies

4.2.1 Step Strategy

One possible strategy is to buy the amount U^{max} for as long as $\mathbf{1}^T u_+ \leq \overline{B}$, and sell on the final days in a similar fashion. This strategy is denoted by u_{step} and is depicted in Figure 4, along with its corresponding profit histogram. It is observed that the histogram is shifted to the left, with an expected profit of \$175, and a standard deviation of \$692. Adopting this strategy results in an inferior profit level compared to the optimal trading strategy.

4.2.2 Linear Strategy

Another possible strategy is to buy constant amounts during the first 30 intervals and sell the same constant amount during the second 30 intervals. In other words, at $t = 30$, the trader's total position is $\overline{B}$. This strategy is denoted by u_{lin} and is depicted in Figure 5,

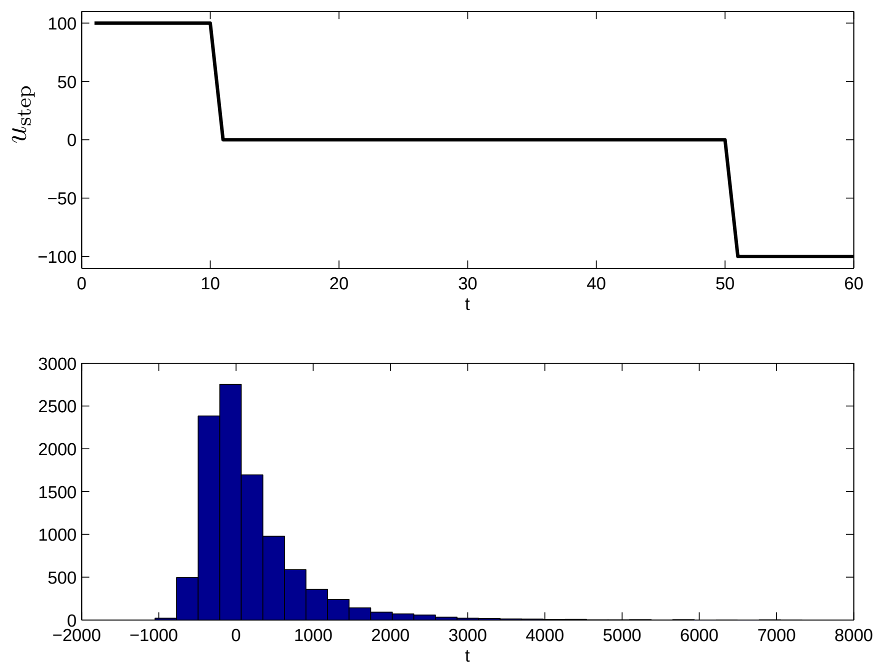

Figure 4: Profit histogram for adopting the step strategy u_{step} (top), and the corresponding profit histogram (bottom).

along with its corresponding profit histogram. It is observed that the histogram is shifted to the left, with an expected profit of \$46, and a standard deviation of \$487. Adopting this strategy also results in an inferior profit level compared to the optimal trading strategy.

4.3 Optimal Strategy with Positive Drift on Price Trajectories

So far we have investigated the case of the neutral market (*i.e.*, price trajectories generated with geometric brownian motion with $\mu = 0$); this implies that the average price is almost constant around p_0 through $t = 1, \ldots, T-1$. In this section, we look at how the optimal trading strategy u^* can change if the market has a drift. We look at the same problem described in §4, but instead use a positive drift ($\mu = 0.025$) in generating the price data for $t = 1, \ldots, T-1$ for the N price trajectories. As in §4, the initial price p_0 and the final price $p_T = kp_0$ are set deterministically. A few example price trajectories and the optimal strategy u_{drift} are depicted in Figure 6. Note that the drift causes the price to shoot up above the final price $p_t = kp_0$. Also, note that since the drift is positive, the optimal strategy is similar to the step strategy, except that no trade takes place in the final time interval (since the average price at time T is lower than the average price at time $T-1$ due to the positive drift).

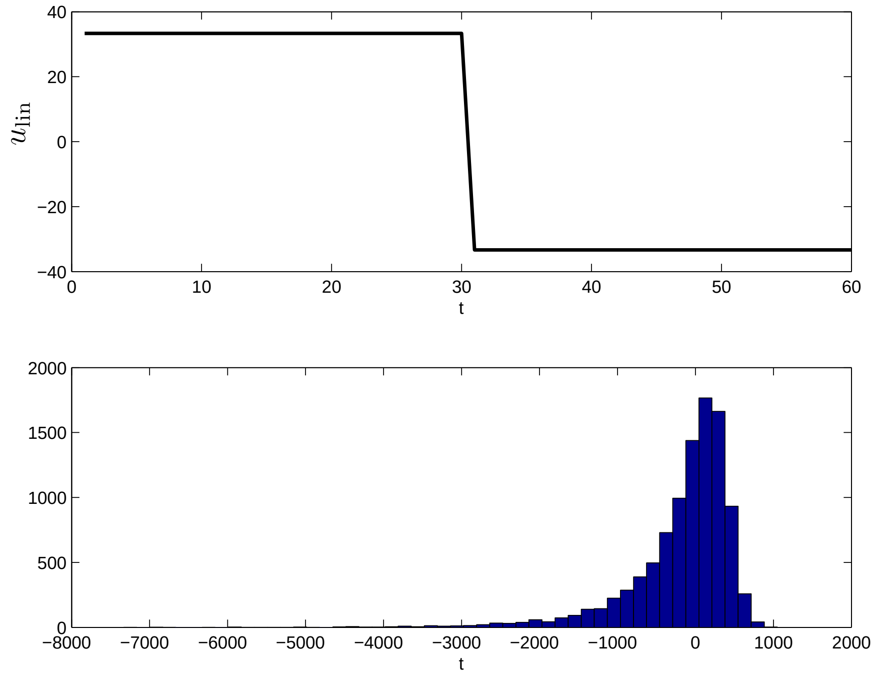

Figure 5: Profit histogram for adopting the linear strategy $u_{\rm lin}$ (top), and the corresponding profit histogram (bottom).

5 Trading Impact

Trading large amounts of shares can affect the price data directly. In order to take this into account, we consider a markov price model similar to [BL98]: $p_t = p_{t-1} + \lambda u_t + \epsilon_t$, where $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$. We solve the problem with $\sigma_\epsilon^2 = 0.01$.

We solve a closed-loop version of this problem using model predictive control, in the following form:

$$\begin{array}{ll} \text{maximize} & \mathbf{E}[-\mathbf{p}^T\mathbf{u} - \kappa\|\mathbf{u}\|_1] \\ \text{subject to} & \mathbf{1}^T\mathbf{u} = 0 \\ & \mathbf{u} \in \mathcal{U} \\ & p_t = p_{t-1} + \lambda u_t + \epsilon_t, \quad t = 1, \ldots, T-1 \\ & p_T = kp_0 \\ & \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)\text{i.i.d.} \end{array} \tag{2}$$

We can solve this using T optimization problems, with the objective of maximizing expected future profit, *i.e.*, $\sum_{\tau=t}^{T} -\kappa|u_t| - p_t u_t$.

The linear impact model introduces a trade-off for the trader: buying more shares results in higher position(which he can hopefully sell at the larger future price; at the same time, this leads to increased current prices, which reduce the trader's profits. We used the same parameters used in §4. Figure 7 shows the optimal trading strategy. It is observed that the

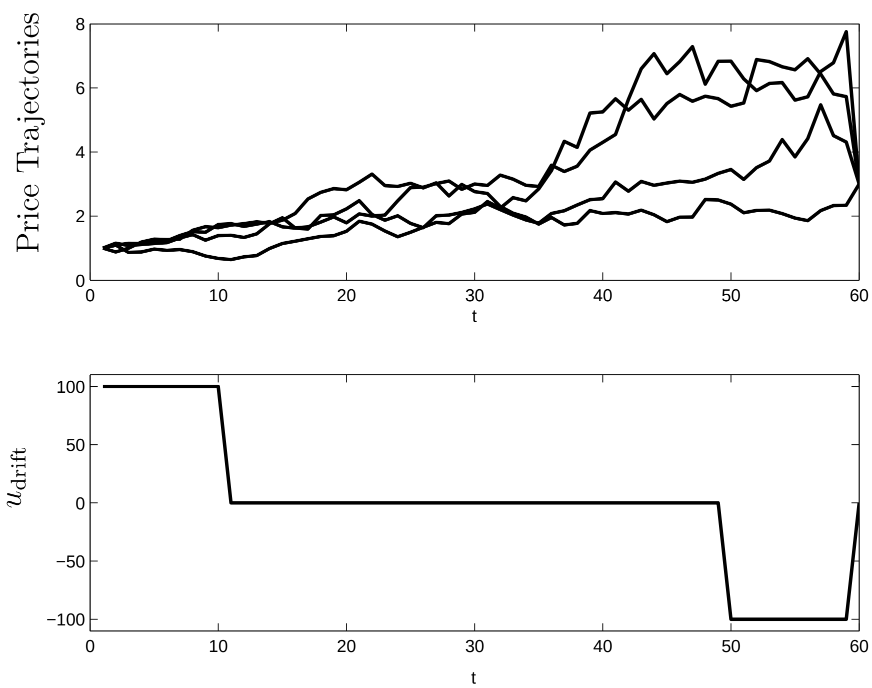

Figure 6: Some example price trajectories with $\mu = 0.025$ (top), and trader's position resulting from adopting the optimal Trading Strategy, u_{drift}(bottom). Note that the optimal strategy buys as many shares as possible in earlier time intervals(when the price is cheaper), and sells the shares towards the end of the period(when the price is higher).

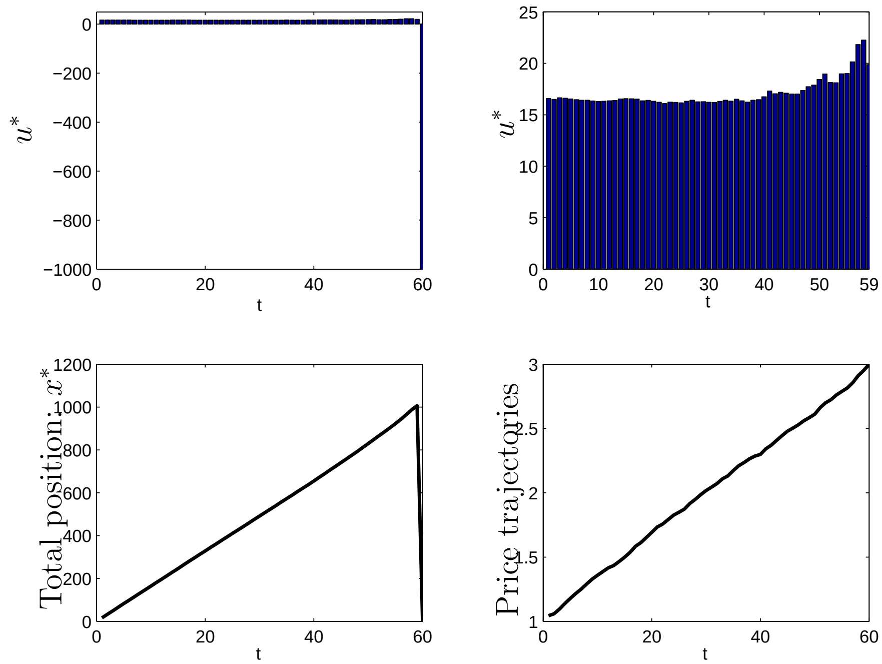

Figure 7: Optimal trading strategy (top left) and the zoomed-in version from $t = 1, \ldots, T-1$ (top right), the resulting position (bottom left), and the evolution of share price (bottom right) with a linear impact model. The optimal strategy is to buy constant amounts of shares and liquidate them all at the final time step.

optimal strategy is to buy almost-constant amounts of shares from $t = 1, \ldots, T-1$ (between \$16-\$19), and liquidate the position at time T. By doing so, the trader gradually drives up the price to kp_0 at time T. This results in an expected overall profit of \$187 and a standard deviation of \$2.06. We compare this strategy to the two strategies suggested in §4.2, the step strategy and the linear strategy. The step strategy results in an objective value of $\$-27.11$, and the linear strategy results in an objective value of $\$-101.4$. This means that adopting these strategies results in a loss rather than a profit, since the trading impact drives the prices up and increases the total cost.

6 Conclusion

In this project, we have computed the optimal trading strategy of an informed trader in the presence of a future positive market shock. In a market where the price follows a geometric brownian motion, the trader's strategy in neutral market conditions is to buy shares right before the occurrence of the shock, and liquidate his position when the shock occurs. However, when the market has a positive drift, the optimal trading strategy is to buy early on when the price is still relatively cheap, and liquidate on the final days before the shock.

When the price of the shares follows a noise-corrupted markov chain, the optimal trading strategy is to buy the same amount throughout the horizon and liquidate when the shock occurs.

7 Future Directions

One possible direction for future work is to consider more complicated price models in order to obtain more realistic strategies. This technique could also be applied to actual price data in order to perform step-by-step calculation of the optimal strategy. Another interesting extension is to consider the problem with the shock happening at a random time in a span of T intervals.

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